Teaching Methods for Secondary Algebra: A Meta-Analysis of Findings

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Algebra student achievement has become a matter for increased focus in recent years, as educators answer calls for reform, including the standards documents of the National Council of Teachers of Mathematics and the No Child Left Behind mandates. A meta-analysis of 35 independent experimental studies shows that six teaching method categories have positive effects on student achievement in the secondary-algebra classroom. Teaching method categories are defined and ranked according to their effect sizes. Recommendations for practitioners are provided.

Student achievement in the secondary-algebra classroom has become a matter for increased focus in recent years, as educators endeavor to provide students essential skills for life in the knowledge-based economy of the 21st century (Midgett & Eddins, 2001). Ongoing reform efforts, including the standards documents of the National Council of Teachers of Mathematics (NCTM), call for robust mathematics curricula, more innovative instructional approaches, and greater access to higher-level mathematics for students traditionally steered in other directions (NCTM, 2000).

Secondary algebra is the fundamental course for students' access to higher-level mathematics. Under the auspices of the No Child Left Behind (NCLB; 2002) mandates, national and state-sponsored standardized testing programs have fostered a results-oriented instructional climate for algebra teachers and school administrators. Faced with a challenge to help all children succeed with the complexities of algebra, instructional leaders are encouraged to turn to research literature for teaching methods that enhance student achievement in secondary algebra.

There are hundreds of research reports written by educators and researchers regarding teaching methods for secondary algebra. While the literature pertaining to this topic abounds with opinion regarding curricula and anecdotal information about instructional techniques and unique classroom situations, there is a body of reports describing careful experimental studies of teaching methods in the secondary-algebra classroom. Generally, these studies each describe an experimental investigation, comparing a treatment group receiving a particular teaching method to a control group in which

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the method is not used. The two groups are then compared based on an outcome variable measuring the participants' algebra knowledge and skills achievement.

A formal approach to reviewing this body of research is necessary, so that educators may make data-driven decisions regarding teaching methodology. Researchers have synthesized results from education studies in order to make broad inferences. Walberg (1984) analyzed an expansive body of studies to determine three main causal influences of student learning: student aptitude, instruction, and environment. Spotlighting instruction, Marzano, Pickering, and Pollock (2001) reviewed research studies of teaching methods to specify and rank, according to effect size, nine instructional techniques that have a positive effect on student achievement across instructional situations. Each of these reviews makes a broad sweep of educational research across all grade levels, student ability levels, and settings in order to draw conclusions regarding student learning.

In an earlier and more focused study, Marcucci (1980) drew on the tenets of Glass (1976) to conduct a meta-analysis of experimental studies of teaching methods to enhance K–12 student achievement in mathematics problem solving. Marcucci reviewed 33 studies conducted from 1950 to 1980, classifying the teaching methods under investigation into four general types: modeling, systematic, heuristic, and guided discovery. Of the four teaching method types, the approach labeled heuristic—a method stressing the teaching of general problem-solving skills such as drawing diagrams or simplifying a problem by using smaller numbers—produced the strongest effects on mathematics student achievement.

Marcucci's (1980) study developed knowledge of teaching methodology for mathematics problem solving considerably. Of the 33 studies integrated by Marcucci, however, only 11 were set in an algebra classroom; thus, he was unable to draw conclusions concerning algebra instruction. Further, since 1980, new technologies and teaching methods have been introduced in the secondary-algebra classroom, allowing for a fresh appraisal of their effects (R. G. Marcucci, personal communication, September 10, 2001). Finally, Marcucci took responsibility for defining teaching method categories and assigning teaching methods to each without a content validation or classification procedure, allowing for researcher bias.

This article reports results from a meta-analysis that builds on Marcucci's (1980) methodology to review and integrate research findings from 1980 to 2002 on teaching methods for secondary-level algebra. This review of literature studies the effects of teaching methods on student achievement on Virginia's End-of-Course Test for Algebra I (Haas, 2002). It is meant to define specific teaching method types and to rank them according to their effect sizes, so that a practitioner may draw from the body of research and use its integrated findings to make pedagogical decisions. Unlike Marcucci's (1980) study, this meta-analysis includes a process for validating the assignment of individual teaching methods into teaching method categories.

Methods

Sources of Data

A representative collection of studies investigating teaching methods in the secondary-algebra classroom was gathered. The collection and selection process began by searching various computer indexes. By using three selection criteria, the initial pool of 1,418 articles was pared down to the final group of 35 studies. To be included in the meta-analysis, a study had to be conducted between 1980 and 2002 at the secondary school level (i.e., grades 7–12) where algebra instruction was the focus. Second, it had to be an experimental investigation, comparing a treatment group to a control group, whose outcome was a measure of the subject's algebra knowledge and skills achievement. Third, the teaching method in the study had to deal with algebra knowledge and skills achievement per se and not with factors related to it.

Study Characteristics

To describe the main features of the 35 studies, eight variables are presented. Two of the variables describe the types of students participating in the studies: ability level of the students as defined by the study's author as unspecified, low, middle, or high; and algebra course level as defined by the researcher as pre-algebra, algebra, or advanced algebra. The next two variables classify the treatment found in each study: length of treatment in terms of weeks and hours and the type of treatment or teaching method under investigation. Two other variables describe study design features: sample size and outcome measure, experimenter designed or standardized. The final two variables describe the source of the study and the year of publication.

Teaching method categories were named and defined on the basis of a review of literature on algebra teaching methods and a careful reading of the studies included in the meta-analysis. Studies were grouped according to the teaching methods under investigation and category names were derived from these groupings. Categories were developed in an attempt to clearly differentiate one type of teaching method from those within other categories. Category definitions and placement of teaching methods under particular categories were tested using a content validation process. During two rounds of content validation, 14 students attending a secondary mathematics teaching methods course at Virginia Polytechnic Institute and State University during the spring of 2002, assessed each teaching method's category placement and strength of association to the category, along with its definition's clarity. (See Haas, 2002, for a more in-depth description of this process.)

Within this process, teaching methods found in nonexperimental research reports were also assessed for appropriate placement under categories in order to augment the categories for further research (Haas, 2002). These nonexperimental methods were not included in the statistical results of the meta-analysis, but they are presented in the results section of this report to give examples of teaching methods for practitioners. When a study seemed to use a combination of teaching methods, the decision for classification into a category was based on the definition of the teaching method and the purpose of the study as described by the researcher. Teaching methods outlined here should not be considered mutually exclusive because one method may contain another. For example, a problem-based learning approach to teaching may include the use of technology, such as calculators for graphing, to enhance student understanding.

It is important to note that the researchers conducting the included studies were responsible for defining teaching methods assigned to their experimental and control groups. For example, Smith (2001) compared achievement of algebra students who received instruction supplemented with the Carnegie Algebra Tutor with those who did not. His study's effect size is included in the mean effect size for the technology-aided instruction teaching method category. Students in Smith's control group may have been subject to any variety of teaching methods with the exception of the Algebra Tutor, including the use of calculators and even other computer software at school or at home.

Such is the case with each study included here. While students in the experimental group for a study included in the direct-instruction category may have received one method such as worked examples, students in the study's control group may have been taught with other direct-instruction methodologies. This is a limitation of meta-analysis that is mitigated somewhat by using mean effect sizes for several studies in a category. The consumer must operate under the assumption that the meta-analysis is just that, an analysis of analyses. It is a broad examination of the strength of individual methods, aggregated into larger categories.

After the conclusion of the second round of content validation, six teaching method categories remained for use as treatment variables:

1. Cooperative learning—Students working together to reach a common goal
2. Communication and study skills—Teaching students to read and study mathematical information effectively and providing opportunities for students to communicate mathematical ideas verbally or in writing (thinking aloud)

3. Technology-aided instruction—Using computer software applications and/or hand-held calculators to enhance instruction
4. Problem-based learning—Teaching through problem solving where students apply a general rule (deduction) or draw new conclusions or rules (induction) based on information presented in the problem
5. Manipulatives, models, and multiple representations—Teaching students techniques for generating or manipulating representations of algebraic content or processes, whether concrete, symbolic, or abstract
6. Direct Instruction—Establishing a direction and rationale for learning by relating new concepts to previous learning, leading students through a specified sequence of instructions based on predetermined steps that introduce and reinforce a concept, and providing students with practice and feedback relative to how well they are doing.

Study Outcomes

The 35 studies describe the effects of six teaching method types on student achievement using assessments of algebra knowledge and skill. To quantify the effects of the teaching method categories, the effect size, defined by Glass (1976) as the difference between treatment and control group mean scores divided by the standard deviation for the control group, was used. Each study was assigned an effect size, and a mean effect size was calculated for each teaching method category as a whole. This allowed for a ranking of teaching method categories according to their effects on student achievement. When studies reported means and standard deviations for both the treatment and control group, effect size was calculated from the scores provided. For studies where these statistics were not reported, effect size was calculated from statistics such as t and F, using procedures delineated by McGaw and Glass (1980).

Results

According to Gay (1996), one characteristic of meta-analysis that distinguishes it from more traditional approaches to research integration is that there is an emphasis for the researcher to be as inclusive as possible. "Reviewers are encouraged to include results typically excluded, such as those presented in dissertation reports and unpublished work" (Gay, 1996, p. 266). Critics have claimed that this emphasis leads to the inclusion of "poor" studies. Glass, McGaw, and Smith (1981) argued that there is no proof that this is the case. They further argued that articles published in journals tend to have larger effect sizes, thus causing an over estimation of the value of an intervention. The inclusion of unpublished reports and dissertations tempers this factor, because dissertations demonstrate higher design quality than published journal articles and tend to have smaller effect sizes.

Table 1 provides a snapshot describing the sample of 35 studies integrated, giving a simple breakdown or comparison between mean effect sizes for studies based on their possession of selected characteristics. Most of the studies analyzed here were completed after 1990. The majority of reports were unpublished papers, such as dissertations, and the unpublished works produced a smaller mean effect size (.36) than those published in journal articles. Those with duration of less than 9 weeks produced a much larger mean effect size, a medium 0.52, than those whose time frame was greater than 9 weeks. Roughly two-thirds of the reports used experimenter-designed outcome measures, yielding a larger mean effect size than for those with standardized tests.

The sample of studies presented in this quantitative review produced a range of effect sizes displayed in Table 2. Research offers three ways to interpret effect size in terms of student achievement. First, effect size may be considered an expression of an increase or decrease in the experimental group's achievement in standard deviation units. A mean effect size of .55 for direct instruction studies represents an average of over one-half of a standard deviation unit increase in achievement for students receiving this treatment. effect size may also be interpreted by translating the expression of standard deviation into a percentile gain or loss. A standard deviation of 1.0, for example, would indicate a 34% difference in scores between the average student in the experimental group and the average student in the control group. Students in the problem-based learning experimental groups experienced a 15% improvement in algebra achievement over students in the control groups. A third way to interpret effect size is offered by Cohen (1988). An effect size of .20 can be considered small, while an effect size of .50 and .80 can be considered medium and large, respectively. Thus, the mean effect sizes for teaching method categories represented in this sample of studies range from very small, for communication and study skills and technology-aided instruction, to medium, direct instruction and problem-based learning.

Teaching method categories varied in their effect on students, depending on the algebra course level and student ability level. Table 3 presents a matrix indicating teaching method type, followed by a statement of effect size according to course level and student ability level. None of the studies analyzed described student ability level as middle or average, and because unspecified ability level could be low, middle, or high, displaying results from this grouping would not indicate results for students of average- or middle-ability level. Only low and high student ability results are presented. A negative effect size indicates that the students in the experimental treatment group scored lower on the outcome measure than those in the control group.

Table 1. Mean and Standard Deviation of Effect Size for Algebra Student Achievement for Selected Study Characteristics

Study Characteristics	Number of Studies (*n* = 35)	Effect Size *M*	Effect Size *SD*
Source of study			
Journal	6	0.41	0.64
Unpublished (dissertation/thesis)	29	0.36	0.58
Duration of study			
Less than 9 weeks	8	0.52	0.15
Greater than 9 weeks	27	0.15	0.60
Outcome measure			
Standardized	11	0.17	0.67
Experimenter-designed	24	0.44	0.55
Year of publication			
1980–1990	14	0.44	0.61
1991–2002	21	0.34	0.57

Table 2. Categories of Teaching Methods Investigated in Secondary Algebra Studies

Category	Number of Studies (*n* = 35)	Number of Effect Sizes	Effect Size *M*	Effect Size *SD*	Percentile Gain[a]	Length of Treatment (weeks) *M*	Length of Treatment (weeks) *SD*	Sample Size *M*	Sample Size *SD*
DI	10	19	0.55	0.59	21	11.68	14.19	83.70	63.30
PB	7	14	0.52	0.70	20	11.00	12.21	188.00	158.30
MR	5	13	0.38	0.43	15	4.80	4.82	132.80	79.08
CL	3	3	0.34	0.16	13	5.67	1.16	145.33	108.30
CS	3	5	0.07	0.26	3	18.00	15.87	102.00	58.59
TA	7	12	0.07	0.69	3	25.57	17.36	168.57	161.17

[a]Percentile gain is determined from a table found in Marzano, Pickering, and Pollock (2001, p. 160).

Direct instruction had the largest effect for low-ability and high-ability students. It is also interesting to note that while problem-based learning had a large effect in the pre-algebra classes studied, it had a small, negative effect in algebra classes and when used with high-ability students. While the results stated here might be idiosyncratic to the sample of studies, it bears mentioning that the studies were comparing student achievement on an assessment and not on in-depth learning per se. It may be that, regardless of ability level, student achievement on an assessment is most positively influenced with a direct instruction approach. The mean length for both direct instuction and problem-based learning studies was approximately 11 weeks. Perhaps 11 weeks is not a long enough time period to assess the benefits of the problem-based learning model in terms of student achievement on tests.

Table 3. Summary of Findings for Teaching Methods' Effects for Student Groups

	Effect Size **(Small, Medium, Large)				
	Course Level			Student Ability	
Teaching Method	Pre-Algebra	Algebra	Advanced Algebra	Low	High
DI	Small	Medium	*	Large	Large
PB	Large	Small Negative	Medium	Medium	Small Negative
MR	Small	Medium	*	*	*
CL	Small	Small	*	*	*
CS	*	Small	Small	*	*
TA	Medium	Small Negative	Medium Small	Small	

* Insufficient data.

**Considered positive unless stated as negative.

Direct Instruction

The direct instruction (DI) teaching method is similar to Marzano, Pickering, and Pollock's (2001) "Setting Objectives and Providing Feedback" category. The authors found an average effect size of .61 across subject areas and grade levels. Two of the 10 DI studies in this meta-analysis were published in journals. Direct instruction produced its largest effect size in algebra course studies (.59) and with students classified as low ability (.84).

DI is a teaching method type that may encompass all of the others. It is a framework for instruction not only used in algebra and other areas of mathematics, but across the curriculum. Tenenbaum's (1986) investigation of enhanced cues, reinforcement, and feedback corrective procedure produced the largest effect size (1.35) of the studies in this category. Table 4 presents a set of specific teaching methods most associated with the DI category in the content validation process. Grading homework to provide feedback was most strongly associated with the definition for direct instruction. When observing a teacher, the administrator should look for instances where the teacher collects homework to provide written feedback to students, rather than simply checking for effort. Specific comments help students to identify error patterns and correct or incorrect thinking. If a teacher indicates that there are too many exercises to "grade" on a typical homework assignment, this may also indicate that there are too many exercises assigned. Practice is important, but practice with feedback is even more beneficial.

Problem-Based Learning

Like direct instruction, problem-based learning (PB) is a teaching method type that may encompass all of the others. It is a framework for instruction not only used in algebra and other areas of mathematics, but across the

Table 4. Teaching Methods Most Strongly Associated With Direct Instruction During Content Validation Study

Method	Author(s) & Year
Grade homework to provide feedback.	Dick, 1980
Close instruction by reviewing concepts with students, emphasizing comparisons to previously covered concepts.	London, 1983
When providing feedback, target incorrect responses and error patterns.	Maccini & Hughes, 2000
Identify a new skill or concept at the beginning of instruction and provide a rationale for learning it.	Maccini & Ruhl, 2000
Provide a graduated sequence of instruction, moving students from concrete to abstract concepts in defined steps.	Maccini & Ruhl, 2000; Hutchinson & Hemingway, 1987
Require students to indicate a one-step-at-a-time process in working equations.	Feigenbaum, 2000
Use pre-worked examples to introduce or reinforce concepts.	Carroll, 1995
When assigning practice work, ensure that the majority of the problems review previously covered material.	Clay, 1998; Pierce, 1984; Denson, 1989

curriculum. PB is similar to Marzano, Pickering, and Pollock's (2001) category labeled "Generating and testing hypothesis," where students apply knowledge to new situations by induction or deduction. The authors found an effect size of .61 across grade levels and subject areas.

PB is most similar to Marcucci's (1980) category, "Guided Discovery," where appropriate questioning strategies are used to guide a student to discover the solution of a problem. Guided discovery teaching methods produced a negative effect size (-0.08) for elementary and secondary students and all courses in Marcucci's study. For the present study, PB had its strongest effect size in the pre-algebra classroom (.74) and with low-ability students (.58). All of the PB studies were unpublished and were completed after the publication of the NCTM's first standards document (1989). In the NCTM's latest standards document (2000), PB is again strongly endorsed:

> Successful problem solving requires knowledge of mathematical content, knowledge of problem-solving strategies, effective self-monitoring, and a productive disposition to pose and solve problems. Teaching problem solving requires even more of teachers, since they must be able to foster such knowledge and attitudes in their students. (p. 341)

Hodgkin's (1994) study of the constructivist teaching method produced the largest effect size for this sample of studies (1.79). Teaching methods most strongly associated with PB are displayed in Table 5. The first specific method

Table 5. Teaching Methods Most Strongly Associated With Problem-Based Learning During Content Validation Study

Method	Author(s) & Year
Have students create their own rules in new problem solving situations.	Davis, 1998
Draw mathematical concepts from "real-life" situations.	Price & Martin, 1997; Makanong, 2000
Have students pursue open-ended and extended problem solving projects.	McIntosh, 1997
Create problems from the interests of individual students.	Choike, 2000; Lwo, 1992; Farrell, 1980
Recognize many alternative problem solving practices.	Brenner et al., 1997; Hodgkins, 1994
Emphasize the problem solving process, rather than the solution.	Brenner et al., 1997
Anchor problem solving skills instruction skills instruction within situations meaningful to the students.	Brenner et al., 1997; Elshafei, 1998
Encourage students to experiment with alternative methods for problem solving.	Carroll, 1995; Wilkins, 1993

listed is of note, because it is a textbook example for the constructivist approach to teaching math, where students would literally construct their knowledge and skills with the teacher's guidance. Overall, these methods represent a student-centered and process-oriented approach to teaching.

Manipulatives, Models, and Multiple Representations

Manipulatives, models, and multiple representations (MR) is similar to Marcucci's (1980) "Modeling" category. Marcucci found a small, negative effect size (-0.17) for modeling. MR is also similar to Marzano, Pickering, and Pollock's (2001) category, "Nonlinguistic Representations," where studies across grade levels and content areas produced a large, positive effect size (.75). The authors included a variety of activities in this category, such as creating graphic representations, making physical models, generating mental pictures, drawing pictures and pictographs, and engaging in kinesthetic activities. For the present study, MR produced the third largest effect size (.38). Four of the five studies quantified were published in journals, while none of the five specified an ability level for students.

The NCTM (2000) included MR within its representation standard, stating that instructional programs should enable students to:

> Create and use representations to organize, record, and communicate mathematical ideas; select, apply and translate among

Table 6. Teaching Methods Most Strongly Associated With Manipulatives, Models, and Multiple Representations During Content Validation Study

Method	Author(s) & Year
Have students use cubes or blocks to represent algebraic equations.	Raymond & Leinenbach, 2000; McClung, 1998; Goins, 2001
Illustrate mathematical concepts for students with pictures.	Price & Martin, 1997
Teach students to represent algebraic equations with graphs.	McCoy, Baker, and Little, 1996; Goins, 2001
Teach students to represent problems with tables.	McCoy, Baker, and Little, 1996; Keller, 1984; St. John, 1992; Goins, 2001
Teach students to represent problems with charts to break information into smaller pieces.	Matthews, 1997
Emphasize the use of multiple representations: words, tables, graphs, and symbols.	Choike, 2000
Provide math games for students to practice algebraic equations.	Tenenbaum, 1986
Use diagrams to help students learn to solve equations.	Austin & Vollrath, 1989; Goins, 2001

> mathematical representations to solve problems; and use representations to model and interpret physical, social, and mathematical phenomena. (p. 360)

A series of specific teaching methods most strongly associated with MR is presented in Table 6. The group of methods offers options for teaching that appeal to tactile and visual learners. The most strongly associated method, using blocks or cubes to represent algebraic equations, is also the most hands-on approach for students. Goins (2001) study of a "visual method" and "manipulatives method" for algebra instruction yielded a near-large effect size (0.73). The visual method consisted of teaching with graphs, diagrams, and tables, while rectangular tile or blocks were used with the manipulatives method. MR posted its largest effects in algebra course studies (0.46).

Cooperative Learning

Cooperative learning (CL) is strongly endorsed by NCTM (2000) and has been widely researched across subject areas. Marzano, Pickering, and Pollock (2001) reported an effect size of .73 for CL across grade levels and subject areas. Marcucci (1980) did not include this category in his meta-analysis due to the paucity of research in this area from 1950 to 1980 (R. G. Marcucci, personal communication, September 10, 2001). Experimental research from

1980 through 2002, focused on CL for algebra instruction, is also apparently minimal. One of the three studies included here was published in a journal. CL produced a mean effect size of .34 for pre-algebra courses, while none of the CL studies in this sample specified ability level for students. The study of peer tutoring with small-group instruction by Hawkins (1982) generated the largest effect size for this sample (.49).

According to Johnson and Johnson (1999), CL must include five elements to enhance student learning: positive interdependence (sense of sink or swim together); face-to-face promotive interaction (helping each other learn, applauding successes and efforts); individual and group accountability (each individual has to contribute to the group achieving its goals); interpersonal and small-group skills (communication, trust, leadership, decision-making, and conflict resolution); and group processing (reflecting on how well the team is functioning and how to function even better). Table 7 offers several examples of teaching methods under the broader category of CL. Note that the method most associated with CL does not involve small groups. Rather, it indicates whole-class collaboration in problem solving.

Communication and Study Skills

The algebra teacher's role in developing communication skills is largely that of providing students with opportunities to read, write, and talk about mathematics in a nonthreatening environment. Further, "teachers must help students to clarify their statements, focus carefully on problem conditions and mathematical explanations, and refine their ideas" (NCTM, 2000, p. 351). Part of this process, as proposed by Pippen and Carr (1989), should be the provision of guidance through directed reading instruction and supplemental reading guides for students as they study word problems and other difficult mathematics literature.

Communication stands alone as a standard in the NCTM's *Principles and Standards for School Mathematics* (2000):

> In high school, there should be a substantial growth in the students' ability to structure logical chains of thought, express themselves coherently and clearly, listen to the ideas of others, and think about their audience when they write or speak. (p. 348)

Study skills instruction was linked with communication skills during the content-validation process. Hodo (1989) defined study skills as special abilities used when studying mathematics. For example, one study skill proposed by Hodo is the practice of "studying graphs, charts, and examples to understand material better" (p. 103). Marcucci (1980) did not include this category in his meta-analysis, as communication and study skills were not widely proposed as part of the mathematics content area prior to the publication of the first

Table 7. Teaching Methods Most Strongly Associated With Cooperative Learning During Content Validation Study

Method	Author(s) & Year
Collaborate with the whole class in finding a solution to a problem.	Davis, 1998
Allow students to engage in cooperative problem solving.	McCoy, Baker, and Little, 1996
Allow students to discuss solutions to algebra problems with peers.	Wilson & Blank, 1999
Allow students to begin homework in class with peer assistance.	Wilson & Blank, 1999
Pair students to work as peer tutors.	Allsop, 1997; Parham, 1994
Reward group performance in the cooperative setting.	Yueh & Alessi, 1988
Assign students to work in homogeneous groups.	Yueh & Alessi, 1988
Assign students to work in heterogeneous groups.	Yueh & Alessi, 1988

NCTM standards document. Experimental research from 1980 through 2002, focused on communication and study skills as a part of algebra instruction, is also apparently minimal. CS produced its largest effect size (.16) in advanced-algebra class studies, while no studies in the sample indicated ability level for students. Research by Pippen and Carr (1989) with reading skills instruction yielded the largest effect size for this sample (.34).

Teaching methods most strongly associated with CS during the content-validation process are presented in Table 8. The general theme among methods is appropriate for this category; students are talking, reading, and writing about algebra. In addition, it is vital for students to learn how to learn about algebra. Mathematics is a purely man-made construct; thus, it does not often come naturally to students. Assistance with approaches to studying algebra is an appropriate use of instructional time and resources.

Technology-Aided Instruction

NCTM (2000) discussed technology as a matter of principle in mathematics instruction, "electronic technologies—calculators and computers—are essential tools for teaching, learning, and doing mathematics. They furnish visual images of mathematical ideas, they facilitate organizing and analyzing data, and they compute efficiently and accurately" (p. 24). Prior to the development of the first personal computers in the early 1980s, computers were not widely used to enhance instruction. Hand-held calculators were primarily used to assist computation and for answer checking; thus, Marcucci (1980) did not include technology-aided instruction (TA) as a category for his meta-

Table 8. Teaching Methods Most Strongly Associated With Communication and Study Skills During Content Validation Study

Method	Author(s) & Year
Encourage students to use mathematics vocabulary terms in class discussions.	Davis, 1998; Feigenbaum, 2000; Gordon, 1988
Have students describe their thought processes orally or in writing during problem solving.	Davis, 1998; Gordon, 1988; Wilson & Blank, 1999
Require students to share their thinking by conjecturing, arguing, and justifying ideas.	McCoy, Baker, & Little, 1996; Wilson & Blank, 1999
Have students write about their problem solving strategies.	McIntosh, 1997
Encourage students to ask questions when difficulties or misunderstandings arise.	Tenenbaum, 1986
Encourage students to explain the reasoning behind their ideas.	Wilson & Blank, 1999
Use reading instructional strategies to help students with comprehension.	Pippen & Carr, 1989
Provide students with study skills instruction.	Hodo, 1989

analysis. Considering the widespread availability of computer and calculator technology and its advocacy for mathematics instruction by NCTM, experimental research in the algebra classroom from 1980 through 2002 is not as abundant as expected. Two of the seven TA studies in the sample were published in a journal.

TA showed its strongest effect size for advanced-algebra course students (1.42), while its second strongest effect size (.63) was found for algebra course students. High-ability students showed the largest ES of .40. TA enhances other teaching methods for algebra by providing opportunities for students to practice algebra skills and visualize algebra concepts, whether through "drill" exercises, "intelligent tutoring," or software applications. Table 9 displays methods for using technology in algebra instruction, including allowing or requiring students to use calculators during testing situations (Hembree & Dessart, 1986). If students are to use calculators for computations and graphing on high-stakes tests, it only makes sense to allow them to use such technology on formative assessments.

Discussion

Glass (1976) said, "A hundred dissertations are mute. Someone must read them and discover what they say" (p. 4). Meta-analysis has the effect of converting the "mute" portion of a body of research studies, the descriptive

Table 9. Teaching Methods Most Strongly Associated With Technology-Aided Instruction During Content Validation Study

Method	Author(s) & Year
Have students use calculators during tests or quizzes.	Hembree & Dessart, 1986
Have students use calculators for problem solving instruction and activities.	Hembree & Dessart, 1986; Heath, 1987
Have students use calculators to help them develop problem solving strategies.	Hembree & Dessart, 1986
Have students use calculators for computations.	Hembree & Dessart, 1986
Have students use graphing calculators to explore linear relationships.	Patterson, 1999
Have students use computer spreadsheets, such as Microsoft Excel, for problem solving instruction.	Sutherland & Rojano, 1993
Assign students to use calculators as a requirement for class participation.	Wilson & Blank, 1999
Use computer software to provide practice opportunities.	Allsop, 1999; Heath, 1987; Geshel-Green, 1986; Wohlgehagen, 1992; Smith, 2001; Thomas & Rickhuss, 1992; Schumacker, Young, & Bembry, 1995

statistics, and communicating it to a consumer in stark terms and without interpretation. Thus, several studies can speak in one clear voice. In this case, the voice is confirming what Marzanno, Pickering, and Pollack (2001) and Walberg (1984) have reported. Instructional choices affect student achievement in measurable terms. The effect sizes for the teaching methods displayed in Table 2 give an order of influence from small to near large for an aggregate of diverse studies with single outcome measures. As the principal and algebra teacher focus on instruction, it would be enlightening for these practitioners to consider the research findings presented here in order to make instructional decisions that optimize their integrated usage of research-based teaching methods.

Recommendations for Practitioners

Sergiovanni and Starratt (1993) discussed teaching effectiveness in terms of teaching as decision making. If educators view teaching effectiveness in terms of the teacher's ability to bring about desired student learning or educational outcomes, then two dimensions of teaching must be considered:

1. The teacher's ability to teach in a way in which learning is viewed by students as meaningful and significant

2. The teacher's ability to adjust teaching strategies as warranted by changes in the teaching and learning situation. (p. 107)

According to Sergiovanni and Starratt (1993), "teaching approaches...seem less an issue of which is the best way than of which is the best way for what purpose" (p.109). The key recommendation to be considered by practitioners in the case of the present study is to consider first the teaching and learning situation. Currently, algebra teachers are faced with a great challenge: teaching all students algebra so that they may successfully complete the algebra course and pass the state and federal government-mandated tests associated with it.

Which teaching methods will produce the strongest influence on algebra student achievement? To prioritize teaching methods based on the results of this study would suggest a recipe for teaching. However, the results do suggest that three teaching methods should be prioritized: direct instruction; problem-based learning; and manipulatives, models, multiple representations.

DI category produced the largest mean effect size (.51) and a percentile gain of 21% for this study. This indicated a medium effect for this teaching method on student achievement in algebra. The approach associated with this method lends itself to teaching a standards-based curriculum: starting with an objective or standard and focusing instruction on it while making assessments of student progress and providing feedback. DI forms a solid foundation for using other strategies.

PB ranked second with a mean effect size (.52) and a percentile gain of 20% for students. This indicated a medium effect for this teaching method on student achievement in algebra. PB activities provide a context for learning and create a situation where students actively construct their knowledge of algebra. When a teacher engages students with contextual problems and encourages multiple approaches and solutions, the teacher is fostering learning with understanding. Algebra knowledge and skills are intended for problem solving; thus, it is sensible to use this mode of teaching in an algebra class.

MR yielded the third largest effect size, near medium (.38), and a percentile gain of 15 points for the average student participating in the aggregated studies' experimental groups. MR enhances other teaching methods for algebra by providing opportunities for students to see and feel algebra on their terms and to communicate it to others in a variety of formats. As we begin to pay closer attention to student learning styles, we should attempt to address those whose strengths are in the visual and tactile realm. Further, if the algebra teacher recognizes that students may learn best by beginning with what is tangible and transitioning to the conceptual, the hands-on approach may facilitate this transition.

It is important not to discount the remaining three teaching methods, as they produced positive effect sizes for this integration of research. When viewed carefully in light of their definitions, each represents less an overarching approach to teaching and more a tool to be incorporated within a lesson. Teachers should possess a wide repertoire of such tools and strategies, given the challenges their students will encounter while learning the concepts and skills presented.

One of the premises of the *Curriculum and Evaluation Standards* of NCTM (1989) is that "instruction in mathematics must be significantly revised" (p. 1). According to Romberg (1998), "Revised instruction implies that classrooms must become discourse communities where conjectures are made, arguments presented, strategies discussed, and so forth" (p. 9). In 1998, Romberg painted a bleak description of the typical mathematics classroom. Instruction follows a pattern involving reviewing homework, explanation and examples of a new problem type by the teacher, seat work where students work independently to practice the new problem type, summarization of work, a question-and-answer period conducted by the teacher, and teacher-assigned homework consisting of similar problems. Invariably, the next class period begins with a review of these homework problems.

Now consider the following more optimistic scenario. The teacher identifies a new skill or concept at the beginning of instruction and provides a rationale for learning it (DI). The teacher then moves on to present the class with instruction and related problem-solving activities derived from real-life situations that are meant to be interesting to the students (PB). In the context of these activities, students use calculators to develop problem-solving strategies or to represent linear relationships in equations with graphs (TA and MR). During the course of the lesson, the teacher encourages students to use mathematics vocabulary, to share their thinking by conjecturing, arguing, and justifying ideas orally or in written form, and to ask questions when difficulties or misunderstandings arise (CS). The class concludes with the teacher providing feedback and reviewing concepts with students, emphasizing comparisons to previously covered concepts. Students do practice work at home that reviews previously covered material as well as the day's target objective (DI).

Final Recommendation

Emphasize DI. Focus on the desired learning outcome and make decisions about pacing and curriculum emphasis so that students may have every opportunity to learn what they are expected to learn. Use teaching methods that suit both the concept and the students who must learn it within the framework of DI. Assess whether students are learning the instructional objectives with methods that are similar to high-stakes tests. Make decisions to adjust teaching methods based on assessment results. NCTM's (2000) assertion regarding effective teaching bears repeating here:

Teachers need several different kinds of mathematical knowledge—knowledge about the whole domain; deep, flexible knowledge about curriculum goals and about the important ideas that are central to their grade level; knowledge about the challenges students are likely to encounter in learning these ideas; knowledge about how ideas can be represented to teach them effectively; and knowledge about how students' understanding can be assessed. (p. 17)

Conclusion

This meta-analysis substantiates some ideas educators have about teaching methods and their role in enhancing student achievement. The practitioner may conclude that teaching method choice makes a difference in student achievement. Further, this study specifies the average strength of different teaching method categories. Working collaboratively, the algebra teacher and supervising administrator can make effective decisions regarding which teaching methods suit both the approach and the student's current level of understanding and achievement.

Using the results from this meta-analysis, the administrator may open a dialogue with algebra teachers regarding their usage of research-based teaching methods. Whether using a formal or informal survey, the administrator can quickly assess which methods teachers are using and take steps to assist them with instruction. These steps may include workshops where algebra teachers are exposed to the research-based methods presented here; reallocation of resources to encourage greater usage of particular methods, including computer and calculator technology and manipulatives; and the development of observation scales or rubrics. For example, the ranking of methods provided in Table 2 can be easily made into a checklist to ascertain how often an algebra teacher or mathematics department is using the most effective, research-based methods. If the administrator and teachers want to assess teaching methodology in more detail, Tables 4–9 would serve to do so.

As principals and assistant principals, administrators often assume that the teachers with whom they work are exposed to research and make research-based pedagogical decisions. Moreover, administrators may take for granted that teachers review experimental research and can translate findings from research studies into practice. Given the high-stakes testing environment in which educators currently work and the need to prepare all students for life and work in the technology-based economy of the present and future, administrators cannot afford to make these assumptions. School administrators, as instructional leaders, can assist algebra teachers in making research-based decisions about teaching methods. This meta-analysis is offered as a resource for support in this process.

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